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Translated by J.J.D.

PMM U.S.S.R.,Vol.47,No.5,pp. 698-700,1983
Printed in Great Britain

0021-8928/83 $10.00+0.00

UDC 532.5

EVOLUTION OF A CONTACT DISCONTINUITY IN THE BAROTROPIC FLOW OF A VISCOUS GAS*

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Single-valued solvability as a whole is established with respect to time for an initial boundary value problem with discontinuity data for the equations of the one-dimensional barotropic flow of a viscous polytropic gas, and the behaviour of the solution is investigated, when the time increases without limit. The line of contact discontinuity is simulated by the trajectory of a piston of small mass located between two gases. In particular, if the discontinuity separates one and the same gas, it is shown that the pressure discontinuity can only disappear in an infinite time, and the discontinuity decays exponentially.

Suppose that at the initial instant $t=0$ the region $-1<\xi<0$ is filled with a gas of viscosity μ_1 with equation of state $p_1=a_1\rho^{\gamma_1}$, and the region $0<\xi<1$ is filled with a gas with corresponding characteristics μ_2 and $p_2=a_2\rho^{\gamma_2}$, where $\mu_i, a_i, \gamma_i>1$ $(i=1,2)$ are positive constants, p is the pressure and ρ is the density. Below, the velocity is denoted by u.

The behaviour of the medium in region $-1<\xi<1$ at $t>0$ is defined as follows. The motion of each gas outside the line of contact discontinuity $\xi=C(t), C(0)=0$ is defined by the equations

$$\rho(u_t+uu_\xi)=\mu u_{\xi\xi}-p_\xi,\ \rho_t+(\rho u)_\xi=0 \tag{1}$$

The conditions of contact discontinuity on the unknown line $\xi=C(t)$ have the form

$$[u]=[\mu u_\xi-p]=0,\ C'(t)=u\ \ ([u]=u(C(t)+0,t)-u(C(t)-0,t)) \tag{2}$$

Further, we will assume that at the points $\xi=-1, \xi=1$ the conditions of adhesion are satisfied

$$u(-1,t)=u(1,t)=0 \tag{3}$$

The functions $u_0(\xi), \rho_0(\xi)$,

$$u(\xi,0)=u_0(\xi),\ \rho(\xi,0)=\rho_0(\xi) \tag{4}$$

that specify the initial conditions are assumed to be smooth when $\xi\neq 0$, while at the point $\xi=0$ the continuity of the functions ρ_0, p_0 is not required.

Problem (1)-(4) is conveniently solved in Lagrangian mass variables

*Prikl.Matem.Mekhan.,47,5,870-872,1983

$$x(\xi, t) = \int_{C(t)}^{\xi} \rho(y, t)\, dy$$

in which the line of contact discontinuity becomes known and has the form $x = 0$.

We will introduce the following notation:

$$\Omega_1 = \{x:\ -h_1 < x < 0\},\ \Omega_2 = \{x:\ 0 < x < h_2\}$$
$$\Omega = \Omega_1 \cup \Omega_2,\ Q_{iT} = \Omega_i \times (0, T),\ \Gamma = \{x, t : x = 0,\ t \geqslant 0\}$$
$$h_i = \left| \int_0^{(-1)^i} \rho_0(\xi)\, d\xi \right|,\ v = \rho^{-1},\ \sigma = \mu\rho u_x - p,\ p = a\rho^{\gamma}$$

where $\mu(x), \gamma(x), a(x)$ are piecewise-constant functions that in the regions Ω_1, Ω_2 take the values μ_1, γ_1, a_1 and μ_2, γ_2, a_2, respectively.

In the new variables the initial problem (1)-(4) is defined as the initial boundary value problem in the region $\Omega \times (t > 0)$ for equations with discontinuous coefficients and initial conditions

$$u_t = \sigma_x,\ v_t = u_x;\ [u]_\Gamma = [\sigma]_\Gamma = 0,\ u|_{\partial\Omega} = 0 \qquad (5)$$
$$u(x, 0) = u_0(x),\ v(x, 0) = v_0(x)$$

We will present the main results using the notation of functional spaces in /1/.

Theorem 1. Let the following conditions be satisfied:

1) $u_0 \in H^{2+\nu}(\bar{\Omega}_i),\ v_0 \in H^{1+\nu}(\bar{\Omega}_i);\ u_0, \sigma_0 \in C(\bar{\Omega});\ 0 < \nu < 1;$
2) $\inf_\Omega v_0 > 0,\ \sup_\Omega v_0 < \infty;$ 3) $\sigma_{0x}|_{\partial\Omega} = 0$

Then a unique classic solution of problem (5) as a whole exists with respect to time, and it has the properties

$$u \in H^{2+\nu,\, 1+\nu/2}(\bar{Q}_{iT}),\ v \in H^{1+\nu,\, 1+\nu/2}(\bar{Q}_{iT}),\ \forall T > 0$$

where the function v is strictly positive and bounded.

Theorem 2. When the time increases without limit the solution of problem (5) is stabilized, i.e. it reduces to a stationary solution $u = 0, p = p_\infty$ in the following sense:

$$\lim_{t\to\infty} \Big\{\sum_1^2 (\|u\|_{W_2^1(\Omega_i)} + \|p - p_\infty\|_{C^1(\Omega_i)}) + |[p]_\Gamma|\Big\} = 0$$

The constant p_∞ is obtained from the relation

$$h_1 (p_\infty a_1^{-1})^{\varkappa_1} + h_2 (p_\infty a_2^{-1})^{\varkappa_2} = \int_\Omega v_0\, dx \equiv \beta, \quad \varkappa = -\gamma^{-1}$$

When $\gamma_1\mu_1^{-1} = \gamma_2\mu_2^{-1}$ and $p_0(0 + 0) \neq p_0(0 - 0)$, positive constants c_i $(i = 1, 2, 3, 4)$ exist independent of t such that

$$c_1 \exp(-c_2 t) \leqslant |[p]_\Gamma| \leqslant c_3 \exp(-c_4 t)$$

Below we give a brief proof of these statements. The solution of problem (5) is obtained as the limit of the solution of the problem (problem A), analogous to (5), as $m \to 0$, where the conditions of contact discontinuity on the line Γ is replaced by

$$[u]_\Gamma = 0,\ mU_t = [\sigma]_\Gamma\ (U(t) = u(0, t))$$

For each fixed $m > 0$ problem A defines the motion of a free piston of mass m, which separates two gases.

Problem A was investigated in /2/, and for it Theorem 1 holds. We pass to the limit on the basis of estimates of the solution, independent of m, in the norms of Hölder spaces and the theorem of imbedding. The method used in /2/ is used to obtain the estimates.

The basis of the proof of Theorem 2 on the stabilization is the estimate of the solutions of problem (5) that are uniform in time. Among these the upper and lower bounds of the density are paramount; they are derived using the laws of conservation of energy and mass

$$\frac{d}{dt}\int_\Omega \left(\frac{1}{2} u^2 + \frac{1}{\gamma - 1} pv\right) dx + \int_\Omega \mu\rho u_x^2\, dx = 0, \quad \frac{d}{dt}\int_\Omega v\, dx = 0$$

from the following relations:

$$p = \frac{\mu}{\gamma}\frac{\partial}{\partial t}\ln\left(1 + \int_0^t E(x, \tau)\, d\tau\right)$$

$$E(x,t)=\frac{\gamma}{\mu}p_0(x)\exp\left\{\frac{\gamma}{\mu\beta}\left[\int_0^t\int_\Omega (u^2+pv)\,dx\,d\tau+B(x,t)\right]\right\}$$

$$B(x,t)=[\mu]\int_0^t U\,d\tau-\int_\Omega v(y,t)\int_y^x u(z,t)\,dz\,dy-\int_\Omega v_0(x)\int_0^x u_0(y)\,dy\,dx+\beta\int_0^x u_0(y)\,dy$$

Once we have time-uniform estimates for the density, all the remaining estimates, as well as the asymptotic forms with respect to time can be obtained by following the same line of reasoning as in /3/.

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Translated by J.J.D.

PMM U.S.S.R.,Vol.47,No.5,pp.700-702,1983
Printed in Great Britain
0021-8928/83 $10.00+0.00

UDC 533.69

A METHOD OF CALCULATING THE AERODYNAMIC CHARACTERISTICS OF BODIES ON THE BASIS OF INVARIANT RELATIONS OF THE THEORY OF LOCAL INTERACTION*

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The existence of relations between the aerodynamic characteristic of solids of revolution of various forms that are invariant to the model of the flow over them and to the angle of attack is proved. A method of calculating the characteristics is developed on that basis. An example of its use for bodies with a generatrix of exponential form is considered, and a comparison with "exact" numerical calculations is made.

Within the framework of models of local interaction (/1-4/ and others) the local interaction force of the flow at each point of the body surface depends only on the local angle of attack and on parameters that define the process of flow over bodies "as a whole". Such models are effectively used over a wide range of flows (the free molecular mode, hypersonic flows of dense and rarefied gas, the light stream flow, and the intermediate region of rarefied gas flow). However, existing methods of aerodynamic calculations (/1,4/ and others) presume a knowledge of the specific model of local interaction.

Let the surface of a convex solid of revolution in the system of coordinates $x\varphi r$ attached to the body be given by the function $r(x)$ with the Ox axis directed along the body axis. An expression for the coefficient of the projection of the aerodynamic force $\mathbf{R}$ on some direction defined by the unit vector $\mathbf{l}$ can be represented in the form

$$C_l=\frac{\mathbf{R}\cdot\mathbf{l}}{qS_k}=\frac{1}{S_k}\int_0^{r_k}\int_0^{2\pi}F_l\left(\alpha,\varphi,\frac{dr}{dx}\right)d\varphi\cdot r\,dr=\int_{u_-}^{u_+}\Phi_l(\alpha,u^{-1})\frac{d}{du}\left(\frac{r}{r_k}\right)^2du,\quad q=\frac{\rho_\infty v_\infty^2}{2}\tag{1}$$

where q is the pressure head, α is the angle of attack, and S_k and r_k are the area and radius of the middle cross section. The functions F_l, Φ_l depend on the indicated arguments and the model of the flow, and $u=dx/dr$ is the cotangent angle of inclination of the body contour to its axis that takes values from u_- to u_+.

Let us consider $n+1$ bodies whose generatrix angle of inclination to the axis varies over the same range. The subscript ν indicates the number of the body. Then, if the function $r_\nu(u)$ $(\nu=0,1,\ldots,n)$ satisfies the condition

$$\left(\frac{r_0}{r_{k0}}\right)^2-\sum_{\nu=1}^n\beta_\nu\left(\frac{r_\nu}{r_{k\nu}}\right)^2=C\tag{2}$$

it follows from (1) that their AX $C_{l\nu}$ of the same kind are connected by the relation

*Prikl.Matem.Mekhan.,47,5,872-874,1983